Part 1: Impact of sidewall Roughness on Integrated Bragg Gratings

Silicon Nanophotonic Course 2012

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AFM measurement shows that the top-down process leaves vertical stripes, which justify the use of the effective index approximation. The sidewall fluctuations follows a normal distribution.


AFM: Atomic force microscopy (the resolution is ~ 0.1-1nm)
Outline

1) Sidewall roughness model

2) Emulator

3) Numerical analysis of apodized uniform gratings

4) Weak grating analysis
Outline

1) Sidewall roughness model

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The sidewall roughness follows a normal distribution and its spectral density function \( G_{\Delta x} \) is obtained with a Fourier transform of its autocorrelation function \( R_{\Delta x} \).

\[
R_{\Delta x} (\Delta z) = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{L/2} \Delta x(z) \Delta x(z + \Delta z) dz
\]

\[
R_{\Delta x} (\Delta z) = \sigma^2 \exp \left( -\frac{\Delta z}{L_c} \right)
\]

\[
G_{\Delta x} (f_z) = \frac{2\sigma^2 L_c}{1 + 4\pi^2 L_c^2 f_z^2}
\]

Typical values: \( \sigma \sim 2 \text{ nm} - 4 \text{ nm} \) // \( L_c \sim 30 \text{ nm}, 300 \text{ nm}, 3000 \text{ nm} \)
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3) Numerical analysis of apodized uniform gratings

4) Weak grating analysis
To analyze the impact of sidewall roughness on IBGs, the roughness perturbation must be transposed to a detuning perturbation.

Waveguide sidewall perturbation ($\sigma$ and $L_c$)
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Waveguide sidewall perturbation ($\sigma$ and $L_c$)

Effective index perturbation ($\sigma_{\text{neff}}$ and $L_c$)

Detuning perturbation ($\sigma_{\delta}$ and $L_c$)

Emulator
To analyze the impact of sidewall roughness on IBGs, the roughness perturbation must be transposed to a detuning perturbation.

- Waveguide sidewall perturbation ($\sigma$ and $L_c$)
  - $\sigma_{neff} \sim \sqrt{2C_{FEM}\sigma}$
  - $\Delta n = C_{FEM}\Delta w$

- Effective index perturbation ($\sigma_{neff}$ and $L_c$)

- Detuning perturbation ($\sigma_\delta$ and $L_c$)

- Emulator
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$\sigma_{\text{neff}} \sim \sqrt{2C_{\text{FEM}}} \sigma$

$\Delta n = C_{\text{FEM}} \Delta w$

Effective index perturbation ($\sigma_{\text{neff}}$ and $L_c$)

$n_{\text{eff}}(\lambda) = n_0 + n_1 \lambda$

$\delta(z) = \frac{2\pi n(z, \lambda)}{\lambda} - \frac{\pi}{\Lambda} - \frac{1}{2} \frac{\partial \theta(z)}{\partial z}$

Detuning perturbation ($\sigma_\delta$ and $L_c$)

Emulator
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Emulator

$$\sigma_{\delta} \approx \frac{\sqrt{2\pi C_{\text{FEM}}}}{n_{\text{ref}} \Lambda}$$

$$\sigma_{\text{neff}} \sim \sqrt{2} C_{\text{FEM}} \sigma$$

$$\Delta n = C_{\text{FEM}} \Delta w$$
The sidewall roughness high frequencies are not relevant to analyze the IBG spectral response, so the Gaussian noise can be filtered a second time by a low-pass filter having $f_{th}$ as a threshold frequency.

\[
\text{randn(length(z),1)/sqrt(mean(diff(z)))}
\]

- Generating a white Gaussian noise (WGN)
- Identify the ideal grating structure ($\delta$ and $\kappa$)

Simulate the noisy grating structure using the couple mode theory
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Generating a white Gaussian noise (WGN)

Identify the ideal grating structure ($\delta$ and $\kappa$)

Filtering with a Lorentzian function having an amplitude of $2L_c \sigma^2 \delta$

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```
randn(length(z),1)/sqrt(mean(diff(z)))
```

- **Generating a white Gaussian noise (WGN)**
- **Identify the ideal grating structure ($\delta$ and $\kappa$)**
- **Filtering with a Lorentzian function having an amplitude of $2L_c\sigma^2_\delta$**
- **Low pass filtering with a threshold frequency of $f_c$**
- **Simulate the noisy grating structure using the couple mode theory**

![Graph showing $G_{\Delta x}/\sigma^2$ vs. $f_z (\mu m)^{-1}$ with different $L_c$ values](image)
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\text{randn(length(z),1)/sqrt(mean(diff(z)))}
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- Generating a white Gaussian noise (WGN)
- Identify the ideal grating structure ($\delta$ and $\kappa$)
- Generating $\delta$ and $\kappa$ vectors with $N > 1/2f_c$
- Filtering with a Lorentzian function having an amplitude of $2L_c\sigma_\delta^2$
- Low pass filtering with a threshold frequency of $f_c$
- Simulate the noisy grating structure using the couple mode theory
The Gaussian noise can be low-pass filtered because high frequencies have out-of-band impact on the IBG spectral response.

\[
n = n(\lambda) + \Delta n \cos \left( \frac{2\pi}{\Lambda} z \right)
\]

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\[ n = n(\lambda) + \Delta n \cos \left( \frac{2\pi}{\Lambda} z + \phi \sin \left( 2\pi f_M z \right) \right) \]

\[ n = n(\lambda) + \Delta n \sum_{m=-\infty}^{\infty} J_m(\phi) \cos \left( \frac{2\pi}{\Lambda} z + m2\pi f_M z \right) \]

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\[ \Delta \lambda = \frac{f_M \Lambda \lambda_B}{1 - 2n_g \Lambda + f_M \Lambda} \]

\[ f_M = \frac{2n_g}{\lambda_B \left( \frac{\lambda_B}{\Delta \lambda} - 1 \right)} \approx \frac{2n_g \Delta \lambda}{\lambda_B^2} \]

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\[ f_c = 30 \times 10^3 \text{ m}^{-1} \]

\[ 2\Delta\lambda = 10 \text{ nm} \]

\[ f_c = 60 \times 10^3 \text{ m}^{-1} \]

\[ 2\Delta\lambda = 20 \text{ nm} \]
Outline

1) Sidewall roughness model

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4) Weak grating analysis
A set of simulation can provide the fraction of gratings that meet specific requirement with the presence roughness. If the yield is not sufficient, a design modification can be done without requiring fabrications and measurements, hence saving time and effort.

$\sigma = 4 \text{ nm, } L_c = 300 \text{ nm}$
A comparison between uniform and Gaussian apodized uniform grating shows that the improvement of the side-lobe suppression ratio is considerably diminished by the roughness.

Unapodized grating
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Unapodized grating

Gaussian-apodized grating
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- Experiments
- Simulations

Unapodized grating

Gaussian-apodized grating
An analytic derivation of the average grating gives fruitful quantitative information of the grating and the noise parameters impact on the IBG spectral response.

\[
    r = - \int_{-L/2}^{L/2} dz \kappa(z)e^{2i\int_0^z dz' \delta(z')}
\]

L : Grating length
An analytic derivation of the average grating gives fruitful quantitative information of the grating and the noise parameters impact on the IBG spectral response.

\[
\begin{align*}
    r &= -\int_{-L/2}^{L/2} dz \kappa(z) e^{2i\int_0^z dz' \delta(z')} \\
    r &= -\int_{-L/2}^{L/2} dz \kappa(z) e^{i\delta(z)} e^{2i\int_0^z dz' \delta(z')}
\end{align*}
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\[
E[rr^*] = E \left[ \int_{-L/2}^{L/2} dz \kappa(z) e^{i \delta_i(z)} e^{2i \int_0^z dz' \delta(z')} \right] \int_{-L/2}^{L/2} d\zeta \kappa(\zeta) e^{-i \delta_i(\zeta)} e^{-2i \int_0^\zeta d\zeta' \delta(\zeta')} \]

\[
E[rr^*] = \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} \left\{ d\zeta \left[ e^{-i \delta_i(\zeta)} e^{i \delta_i(z)} \right] \kappa(\zeta) e^{-2i \int_0^\zeta d\zeta' \delta(\zeta')} \kappa(z) e^{2i \int_0^z dz' \delta(z')} \right\} 
\]

\[
E[rr^*] \approx \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} \left\{ d\zeta \left(1 + E\left[ \delta_i(z) \delta_i(\zeta) \right]\right) \kappa(\zeta) e^{-2i \int_0^\zeta d\zeta' \delta(\zeta')} \kappa(z) e^{2i \int_0^z dz' \delta(z')} \right\} 
\]
An analytic derivation of the average grating gives fruitful quantitative information of the grating and the noise parameters impact on the IBG spectral response.

\[
\begin{align*}
\mathbf{r} &= - \int_{-L/2}^{L/2} dz \kappa(z)e^{2i\int_{0}^{z} dz'\delta(z')} \\
\mathbf{r} &= - \int_{-L/2}^{L/2} dz \kappa(z)e^{i\delta(z)}e^{2i\int_{0}^{z} dz'\delta(z')} \\
E[rr^*] &= E\left[ \int_{-L/2}^{L/2} dz \kappa(z)e^{i\delta(z)}e^{2i\int_{0}^{z} dz'\delta(z')} \int_{-L/2}^{L/2} d\zeta \kappa(\zeta)e^{-i\delta(\zeta)}e^{-2i\int_{0}^{\zeta} d\zeta'\delta(\zeta')} \right] \\
E[rr^*] &\approx \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} d\zeta \left\{ d\zeta \left( 1 + E\left[ \delta(z)\delta(\zeta) \right] \right) \kappa(\zeta)e^{-2i\int_{0}^{\zeta} d\zeta'\delta(\zeta')} \kappa(z)e^{2i\int_{0}^{z} dz'\delta(z')} \right\} \\
E[rr^*] &\approx R + R_n
\end{align*}
\]

$L$: Grating length
The noise contribution is linearly proportional to L and $L_c$, but has a quadratic dependency in function of $\kappa L$, $\sigma$ and $C_{FEM}$.

$$E[rr^*] \approx R + R_n$$

$$R_n = 2L_c \left( \frac{2\sqrt{2\pi}C_{FEM} \sigma}{n_{ref} \Lambda} \right)^2 \frac{\kappa^2 L}{2} \frac{1}{\delta^2} \left[ \cos^2 (L\delta) - \frac{\sin (2L\delta)}{2L\delta} \right]$$
The noise contribution is linearly proportional to $L$ and $L_c$, but has a quadratic dependency in function of $\kappa L$, $\sigma$ and $C_{FEM}$.

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$$R_n = 2L_c \left( \frac{2\sqrt{2\pi}C_{FEM}\sigma}{n_{ref}\Lambda} \right)^2 \frac{\kappa^2 L}{2} \frac{1}{\delta^2} \left[ \cos^2 \left( L\delta \right) - \frac{\sin \left( 2L\delta \right)}{2L\delta} \right]$$
To prove the validity of this derivation, the mean of a thousand simulated uniform gratings is compared to the analytic expression of $E[rr^*]$. 

- a) $L = 1$ mm, $\kappa L = 0.8$, $L_c = 20$ nm, $\sigma = 1.5$ nm
- b) $L = 2.8$ mm, $\kappa L = 0.8$, $L_c = 20$ nm, $\sigma = 1.5$ nm
- c) $L = 2.8$ mm, $\kappa L = 0.8$, $L_c = 60$ nm, $\sigma = 1.5$ nm
- d) $L = 2.8$ mm, $\kappa L = 0.8$, $L_c = 60$ nm, $\sigma = 4$ nm
However, to analyze the spectral distortion, it is the variance of the spectral response that should be analysed.

\[
\sigma_R^2 = E[rr^*rr^*] - \left( E[rr^*] \right)^2
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\[
\sigma_R^2 = E[rr^* rr^*] - \left(E[rr^*]\right)^2
\]

\[
E[rr^* rr^*] = E\left[\int_{-L/2}^{L/2} dz e^{i\delta(z)} \kappa e^{2i\delta_d z} \int_{-L/2}^{L/2} dz' e^{-i\delta(z')} \kappa e^{-2i\delta_d z'} \right]
\]

\[
\approx \Lambda
\]
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\[
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\]
However, to analyze the spectral distortion, it is the variance of the spectral response that should be analysed.

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\[
\int_{-L/2}^{L/2} d\zeta e^{i\delta_i(\zeta)} \kappa e^{2i\delta_d \zeta} \int_{-L/2}^{L/2} d\zeta' e^{-i\delta_j(\zeta')} \kappa e^{-2i\delta_d \zeta'}
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However, to analyze the spectral distortion, it is the variance of the spectral response that should be analysed.

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\]

\[
\max(\sigma_R) \approx \sqrt{\frac{L_cL}{n_{ref}\Lambda}} \frac{C_{FEM}}{(\kappa L)^2}
\]

a) L = 1 mm, \kappa L = 0.8, L_c = 20 nm, \sigma = 1.5 nm

b) L = 2.8 mm, \kappa L = 0.8, L_c = 20 nm, \sigma = 1.5 nm
What’s next?

\[ \max (\sigma_R) \approx \frac{C_{FEM}}{n_{ref}} \frac{\sigma}{\Lambda} (\kappa L)^2 \sqrt{L_c L} \]
max(σ_r) \approx \frac{C_{FEM} \sigma}{n_{ref} \Lambda} (KL)^2 \sqrt{L_c L}
What’s next?

\[
\max \left( \sigma_R \right) \approx \frac{C_{FEM} \sigma}{n_{ref} \Lambda} \left( \kappa L \right)^2 \sqrt{L_c L}
\]
What’s next?

\[
\max (\sigma_R) \approx \frac{C_{FEM}}{n_{ref} \Lambda} \frac{\sigma}{(\kappa L)^2} \sqrt{L_c L}
\]

Conclusion

• We examined the importance of sidewall roughness on IBGs spectral response. We proposed a technique to emulate IBG spectral responses in the presence of an imperfect waveguide.

• We presented an analytic study of sidewall roughness. This analysis shows why the noise has a relatively small impact on the reflection strength at the Bragg wavelength while larger distortions appear on each side of the main reflection peak.

• We have shown that the noise contribution to the grating distortion increases as the square of κL, linearly with σ and $C_{\text{FEM}}$, and as a square root of L and $L_c$. 
Part 2: Characterization of Integrated Bragg Gratings

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Due to the sidewall roughness, the design of “long” integrated Bragg gratings (IBG) on Silicon-on-Insulator (SOI) must be done carefully.

The use of hybrid multimode/singlemode waveguide considerably decrease the phase noise caused by sidewall roughness.
1) Spectral Measurement
Outline

1) Spectral Measurement

2) Grating Reconstruction
Outline

1) Spectral Measurement

2) Grating Reconstruction

3) Silicon layer thickness characterization
Outline

1) Spectral Measurement
2) Grating Reconstruction
3) Silicon layer thickness characterization
4) Apodisation Characterization
Outline

1) Spectral Measurement

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4) Apodisation Characterization
To analyze IBGs reflection spectrum, a time filtering must be done to suppress the interference with spurious reflections.

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Outline

1) Spectral Measurement

2) Grating Reconstruction

3) Silicon layer thickness characterization

4) Apodisation Characterization
An integral layer peeling algorithm was applied to the complex reflection spectrum of the measured IBG.
An integral layer peeling algorithm was applied to the complex reflection spectrum of the measured IBG

Only the low spatial frequencies have an impact on the grating spectral response. The high frequency components are only noise...

\[
n = n(\lambda) + \Delta n \cos\left(\frac{2\pi}{\Lambda} z + \phi \sin\left(2\pi f_M z\right)\right)
\]

\[
n = n(\lambda) + \Delta n \sum_{m=\infty}^{\infty} J_m(\phi) \cos\left(\frac{2\pi}{\Lambda} z + m2\pi f_M z\right)
\]

\[
\Delta \lambda = \frac{f_M \Lambda \lambda_B}{1 - 2n_g \Lambda + f_M \Lambda}
\]

\[
f_M = \frac{2n_g}{\lambda_B \left(\frac{\lambda_B}{\Delta \lambda} - 1\right)} \approx \frac{2n_g \Delta \lambda}{\lambda_B^2}
\]

Since high spatial frequencies have an out-of-band impact on the IBG spectral response, the spatial profile must be low-pass filtered.

\[ f_c \approx \frac{2n_g \Delta \lambda}{\lambda_B^2} \]

**Measurement noise level**

- **a)** \( f_c = 30 \times 10^3 \text{ m}^{-1} \)
  - \( 2\Delta\lambda = 10 \text{ nm} \)
  - Measurement noise level: \(-35 \text{ dB} \)

- **b)** \( f_c = 60 \times 10^3 \text{ m}^{-1} \)
  - \( 2\Delta\lambda = 20 \text{ nm} \)
  - Measurement noise level: \(-40 \text{ dB} \)
Since high spatial frequencies have an out-of-band impact on the IBG spectral response, the spatial profile must be low-pass filtered.
Since high spatial frequencies have an out-of-band impact on the IBG spectral response, the spatial profile must be low-pass filtered.

\[ f_c = 3 \times 10^3 \text{ m}^{-1} \]
Outline

1) Spectral Measurement

2) Grating Reconstruction

3) Silicon layer thickness characterization

4) Apodisation Characterization
The use of hybrid singlemode/multimode IBG strongly reduces the phase noise caused by sidewall roughness which allows characterizing the low frequency components of the thickness of the silicon layer.

\[ \Delta n = \frac{\Delta \lambda_B}{2\Lambda} \]
The use of hybrid singlemode/multimode IBG strongly reduces the phase noise caused by sidewall roughness which allows characterizing the low frequency components of the thickness of the silicon layer.

\[
\Delta n = \frac{\Delta \lambda_B}{2 \Lambda} = C_{WTV} \Delta h
\]

Obtained using a Finite element mode solver
The use of hybrid singlemode/multimode IBG strongly reduces the phase noise caused by sidewall roughness which allows characterizing the low frequency components of the thickness of the silicon layer

\[
\Delta n = \frac{\Delta \lambda_B}{2 \Lambda} = C_{WTV} \Delta h
\]

\[
\Delta h = \frac{\lambda_B(z) - \bar{\lambda}_B}{2 \Lambda C_{WTV}}
\]

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\[ \Delta n = \frac{\Delta \lambda_B}{2\Lambda} = C_{WTV} \Delta h \]

\[ \Delta h = \frac{\left( \lambda_B(z) - \overline{\lambda_B} \right)}{2\Lambda C_{WTV}} \]

Obtained using a Finite element mode solver

\[ \sigma = 0.11 \text{ nm} \]
Outline

1) Spectral Measurement

2) Grating Reconstruction

3) Silicon layer thickness characterization

4) Apodisation Characterization
We tested two approaches that controlled the apodization profile while letting unchanged the effective index.

\[ n = n_{\text{eff}}(\lambda) + \Delta n_{\text{eff}} f(z) \cos \left( \frac{2\pi}{\Lambda} z + \theta(z) \right) \]

1) Superposition apodization

We tested two approaches that controlled the apodization profile while letting unchanged the effective index.

\[ n = n_{eff}(\lambda) + \Delta n_{eff} f(z) \cos \left( \frac{2\pi}{\Lambda} z + \theta(z) \right) \]

1) Superposition apodization

\[ n = n_{eff}(\lambda) + \frac{\Delta n_{eff,LG}}{2} \cos \left( \frac{2\pi}{\Lambda_{LG}} z + \theta_{LG}(z) + F(z) \right) + \frac{\Delta n_{eff,RG}}{2} \cos \left( \frac{2\pi}{\Lambda_{RG}} z + \theta_{RG}(z) - F(z) \right) \]

\[ n = n_{eff}(\lambda) + \Delta n_{eff} \cos \left( F(z) \right) \cos \left( \frac{2\pi}{\Lambda} z + \theta(z) \right) \]

\[ F(z) = \arccos \left( f(z) \right) \]

We tested two approaches that controlled the apodization profile while letting unchanged the effective index.

\[ n = n_{\text{eff}}(\lambda) + \Delta n_{\text{eff}} f(z) \cos\left(\frac{2\pi}{\Lambda} z + \theta(z)\right) \]

2) Phase apodization

\[ n = n_{\text{eff}} + \Delta n_{\text{eff}} \cos\left(\frac{2\pi}{\Lambda} z + \theta(z) + \phi(z) \sin(\omega_M z)\right) \]

\[ n = n_{\text{eff}} + \Delta n_{\text{eff}} \sum_{m=-\infty}^{\infty} J_m(\phi(z)) \cos\left(\frac{2\pi}{\Lambda} z + m\omega_M z + \theta(z)\right) \]

We tested two approaches that controlled the apodization profile while letting unchanged the effective index.

\[ n = n_{\text{eff}}(\lambda) + \Delta n_{\text{eff}} f(z) \cos \left( \frac{2\pi}{\Lambda} z + \theta(z) \right) \]

2) Phase apodization

We tested two approaches that controlled the apodization profile while letting unchanged the effective index.

\[ n = n_{\text{eff}}(\lambda) + \Delta n_{\text{eff}} f(z) \cos \left( \frac{2\pi}{\Lambda} z + \theta(z) \right) \]

2) Phase apodization

We successfully characterized the precision of those two apodisation techniques

1) Superposition apodization

2) Phase apodization
Conclusion

• We extracted the IBG spectral responses by time windowing of the device impulse response thereby eliminating unwanted broadband reflections.

• We showed that the filtering of high spatial frequencies of the reconstructed profiles was appropriate for noise suppression.

• Using the phase information of the grating, we examined the silicon layer thickness variation.

• Using the amplitude information of the grating, we examined the precision of different apodization technique